# appendix a

# APPENDIX A

## STATISTICAL TESTS FOR INDEPENDENCE, TREND, HOMOGENEITY AND RANDOMNESS (1)

### INTRODUCTION

Statistical frequency analysis assumes that the sample to be analysed is a reliable set of measurements of independent random events from a homogeneous population, and the validity of this assumption can be verified using statistical significance tests. The Environment Canada program - NONPARA was used to test the assumptions for the Ottawa River data described in Section 2.0.

### DESCRIPTIONS OF THE TESTS

Brief descriptions of the rationale for each test are given here and illustrated by referring to examples of output; fuller descriptions are given by the Natural Environment Research Council (1975) and Seigel (1956). The theory of the tests is not given but the required functions to be evaluated and the determination of their significance are given in Appendix B pages B1 to B7.

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(1) Adapted from "Statistical Tests for Independence, Trend and Homogeneity and Randomness" by Shin-Young Siau and R. Condie, Hydrologic Applications Division; Water Resources Branch, Inland Waters Directorate, Environment Canada, Ottawa, Ontario, September, 1980.

(1) Spearman rank order serial correlation coefficient test for independence.

Two events can be considered independent only if the probability of occurrence of either is unaffected by the occurrence of the other, and this definition can be extended to a sample of size N. Practically, in a time series, independence can be measured by the significance of the correlation coefficient between the N-1 pairs of the ith and (i+1)th members of the series and if the correlation coefficient is not significantly greater than zero, then independence is assumed. It is noted here that, in the strict mathematical sense, this does not necessarily define independence. To avoid the assumptions made in the derivation of the sampling distribution of the Pearson correlation coefficient, the nonparametric Spearman rank order serial correlation coefficient is used.

Referring to the first page of output for the Ottawa River at Chats Falls, page A-7, the third column shows the data in chronological order, and the fourth column shows the ranking within the series of length $N_1 = 36$. The Spearman rank order serial correlation coefficient is computed between the ranks for 1915-1916, 1916-1917, 1917-1918, etc., using the methods shown on pages B(2) to B(3), which also gives the method determining its significance.

(2) Spearman rank order correlation coefficient test for trend.

If successive measurements of members of a time series have been made during a period of gradually changing conditions, then there will be a more or less noticeable trend in the magnitude of the members of the series when arranged in chronological order. As an example from hydrology, it would be expected that gradual land use changes in a drainage basin would affect the magnitude of the annual flood. Similarly, long-term climatic changes will be reflected in the hydrology of a basin, although it is customary to assume climatic time invariance.

A very simple test for presence of trend and its significance can be made on the Spearman rank order correlation coefficient. Referring again to the Ottawa River at Chats Falls, page A-7, the members of the data series are in chronological order and their rankings described in column 4 and column 6 gives the sequential year number starting with rank 1 for 1915. The Spearman rank order correlation coefficient is then computed for the pairs of entries in columns 4 and 6. The computation and determination of its significance is given in Appendix B, page B(4).

(3) Mann-Whitney split sample test for homogeneity.

If some more or less abrupt change occurred during the sampling period, then some difference could be expected between the means of the sub-samples before and after the change. Examples from hydrology could include the construction of a reservoir in the basin, or a forest fire which denuded a substantial portion of the basin. Assuming a normal distribution, and that the two sub-samples have the same variance then the difference in the sub-sample means can be tested for significance using the distribution of Student's t. These assumptions are not commonly met in hydrology and so the Mann-Whitney non-parametric test is used. Once again, using the Ottawa River at Chats Falls as an example, page A-7, columns 8 and 9 show respectively the annual maximum flood and its ranking, and the sample has been split into sub-samples 1 and 2. With two sub-samples of approximately the same size, it would be expected that if there were no change in conditions, the sums of the ranks of the two sub-samples would not differ significantly. Here, the sums of ranks of the sub-samples are 1192 and 1019, and the question to be answered is whether the difference is significant or not. The Mann-Whitney U statistic is a function of the sub-sample sizes and their sums of ranks. The distribution of U is known and critical values at various levels of significance have been tabulated. Hence, a decision can be made on whether the means of the sub-samples differ significantly. Computation methods are shown in Appendix B, page B(5).

(4) Wald-Wolfowitz split sample test for homogeneity.

This test can determine whether two samples have significantly different means, variances, skewness or kurtosis but it is not as powerful as the Mann-Whitney test in detecting a significant difference in means. On the other hand, the Mann-Whitney test cannot detect differences between the other statistical parameters of the two sub-samples. Referring to the first page of output of any of the examples, column 10 ranks the entire sample in descending order and column 11 determines the sub-sample from which the corresponding data item came. If the sample is well-mixed, the 1's and 2's will be well mixed and the number of runs will be relatively high. A run is defined as a sequence of identical symbols preceded and followed by a different symbol or no symbol at all. Consider the case where the mean of sub-sample 2 is significantly greater than that of sub-sample 1, then 2's will tend to cluster towards the top end of the scale and 1's toward the bottom, thus reducing the number of runs. Suppose now that the variance of sub-sample 2 were substantially greater than that of sub-sample 1. The 2's will then tend to cluster towards both ends of the scale, again reducing the number of runs. Similarly, differences in skewness and kurtosis will cause clustering, reducing the number of runs. The distribution of the number of runs is known and lower critical values have been tabulated. Thus, it can be determined if the sub-samples differ significantly in any respect.

Like the Mann-Whitney test, the Wald-Wolfowitz test can be used to decide whether hydrologic events occurring in different seasons differ significantly. The test is illustrated using the annual flood series on the Ottawa River at Chats Falls, pages A-7 and A-8.

(5) Runs above and below the median for general randomness.

This is a very simple non-parametric test and its operation can be explained by referring to the first page of output of any of the examples. Data are ranked in chronological order, column 3, and the median determined. Column 13 shows an A or B or * according to whether the corresponding data item is above or below or equal to the median, and Column 14 indicates the number of runs. Theoretically, the number of runs, RUNAB, could be as high as N, indicating an extreme short-term cyclic pattern, or as low as 2, indicating an abrupt change half way through the period over which the sample was collected. Notice that the median is used since the probability of exceeding the median is always 0.5, regardless of the probability distribution from which the sample was drawn, thus making the test non-parametric or distribution free. The distribution of RUNAB is known and upper and lower critical values have been tabulated, thus enabling a decision to be made on whether the data are random or not.

TESTS FOR INDEPENDENCE, TREND, HOMOGENEITY AND RANDOMNESS

OTTAWA RIVER AT CHAT FALLS UAN STN. NO. 02KF009
ANNUAL MAXIMUM DAILY FLOW SERIES 1915 TO 1980 DRAINAGE AREA= 89600.0 SQ.MI.

| MONTH | YEAR | DATA | RANK | DI | SEQUENTIAL YEAR NO. | DT | DATA | RANK | ORDERED | FROM SUBSAMPLE | RUNS | ABOVE OR BELOW MEDIAN | RUNAB | YEAR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | SUBSAMPLE 1 | SAMPLE SIZE = 36 | | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| 5 | 1915 | 1630. | 66.0 | 48.0 | 1 | 65.0 | 1630. | 66.0 | 5800. | 2. | 1. | B | 1. | 1915 |
| 5 | 1916 | 4080. | 18.0 | 20.0 | 2 | 16.0 | 4080. | 18.0 | 5750. | 1. | 2. | A | 2. | 1916 |
| 5 | 1917 | 3140. | 38.0 | 18.0 | 3 | 35.0 | 3140. | 38.0 | 5550. | 2. | 3. | B | 3. | 1917 |
| 5 | 1918 | 2401. | 56.0 | 48.0 | 4 | 52.0 | 2401. | 56.0 | 5410. | 2. | 3. | B | 3. | 1918 |
| 5 | 1919 | 4790. | 8.0 | 52.0 | 5 | 3.0 | 4790. | 8.0 | 5070. | 1. | 4. | A | 4. | 1919 |
| 5 | 1920 | 2250. | 60.0 | 33.0 | 6 | 54.0 | 2250. | 60.0 | 4840. | 1. | 4. | B | 5. | 1920 |
| 5 | 1921 | 3510. | 27.0 | 22.0 | 7 | 20.0 | 3510. | 27.0 | 4810. | 2. | 5. | A | 6. | 1921 |
| 4 | 1922 | 5070. | 5.0 | 20.0 | 8 | 3.0 | 5070. | 5.0 | 4790. | 1. | 6. | A | 6. | 1922 |
| 5 | 1923 | 3620. | 25.0 | 18.0 | 9 | 16.0 | 3620. | 25.0 | 4670. | 1. | 6. | A | 6. | 1923 |
| 5 | 1924 | 3030. | 43.0 | 11.0 | 10 | 33.0 | 3030. | 43.0 | 4560. | 1. | 6. | B | 7. | 1924 |
| 5 | 1925 | 2490. | 54.0 | 13.0 | 11 | 43.0 | 2490. | 54.0 | 4500. | 1. | 6. | B | 7. | 1925 |
| 5 | 1926 | 3090. | 41.0 | 14.0 | 12 | 29.0 | 3090. | 41.0 | 4470. | 1. | 6. | B | 7. | 1926 |
| 4 | 1927 | 2440. | 55.0 | 53.0 | 13 | 42.0 | 2440. | 55.0 | 4420. | 1. | 6. | B | 7. | 1927 |
| 5 | 1928 | 5750. | 2.0 | 10.0 | 14 | 12.0 | 5750. | 2.0 | 4280. | 2. | 7. | A | 8. | 1928 |
| 5 | 1929 | 4470. | 12.0 | 32.0 | 15 | 3.0 | 4470. | 12.0 | 4220. | 1. | 8. | A | 8. | 1929 |
| 7 | 1930 | 2971. | 44.0 | 19.0 | 16 | 28.0 | 2971. | 44.0 | 4160. | 1. | 8. | B | 9. | 1930 |
| 4 | 1931 | 2010. | 63.0 | 24.0 | 17 | 46.0 | 2010. | 63.0 | 4110. | 2. | 9. | B | 9. | 1931 |
| 11 | 1932 | 3111. | 39.0 | 24.0 | 18 | 21.0 | 3111. | 39.0 | 4080. | 1. | 10. | B | 9. | 1932 |
| 4 | 1933 | 4220. | 15.0 | 2.0 | 19 | 4.0 | 4220. | 15.0 | 3910. | 2. | 11. | A | 10. | 1933 |
| 5 | 1934 | 4420. | 13.0 | 45.0 | 20 | 7.0 | 4420. | 13.0 | 3821. | 2. | 11. | A | 10. | 1934 |
| 5 | 1935 | 2310. | 58.0 | 48.0 | 21 | 37.0 | 2310. | 58.0 | 3820. | 2. | 11. | B | 11. | 1935 |
| 5 | 1936 | 4560. | 10.0 | 21.0 | 22 | 12.0 | 4560. | 10.0 | 3790. | 2. | 11. | A | 12. | 1936 |
| 5 | 1937 | 3430. | 31.0 | 15.0 | 23 | 8.0 | 3430. | 31.0 | 3770. | 2. | 11. | A | 12. | 1937 |
| 4 | 1938 | 4160. | 16.0 | 8.0 | 24 | 8.0 | 4160. | 16.0 | 3650. | 1. | 12. | A | 12. | 1938 |
| 5 | 1939 | 3650. | 24.0 | 5.0 | 25 | 1.0 | 3650. | 24.0 | 3620. | 1. | 12. | A | 12. | 1939 |
| 6 | 1940 | 3451. | 29.0 | 18.0 | 26 | 3.0 | 3451. | 29.0 | 3540. | 2. | 13. | A | 12. | 1940 |
| 4 | 1941 | 4500. | 11.0 | 24.0 | 27 | 16.0 | 4500. | 11.0 | 3510. | 1. | 14. | A | 12. | 1941 |
| 4 | 1942 | 3230. | 35.0 | 26.0 | 28 | 7.0 | 3230. | 35.0 | 3480. | 2. | 15. | B | 13. | 1942 |
| 5 | 1943 | 4670. | 9.0 | 41.0 | 29 | 20.0 | 4670. | 9.0 | 3451. | 1. | 16. | A | 14. | 1943 |
| 5 | 1944 | 2570. | 50.0 | 13.0 | 30 | 20.0 | 2570. | 50.0 | 3450. | 1. | 16. | B | 15. | 1944 |
| 6 | 1945 | 3170. | 37.0 | 22.0 | 31 | 6.0 | 3170. | 37.0 | 3430. | 1. | 16. | B | 15. | 1945 |
| 4 | 1946 | 2260. | 59.0 | 53.0 | 32 | 27.0 | 2260. | 59.0 | 3280. | 2. | 17. | B | 15. | 1946 |
| 6 | 1947 | 4840. | 6.0 | 45.0 | 33 | 27.0 | 4840. | 6.0 | 3270. | 2. | 17. | A | 16. | 1947 |
| 4 | 1948 | 2560. | 51.0 | 21.0 | 34 | 17.0 | 2560. | 51.0 | 3231. | 2. | 17. | B | 17. | 1948 |
| 4 | 1949 | 3450. | 30.0 | 22.0 | 35 | 5.0 | 3450. | 30.0 | 3230. | 1. | 18. | A | 18. | 1949 |
| 5 | 1950 | 2550. | 52.0 | 49.0 | 36 | 16.0 | 2550. | 52.0 | 3171. | 2. | 19. | B | 19. | 1950 |
| | | | | | | | SUBSAMPLE 2 | SAMPLE SIZE = 30 | | | | | | |
| 4 | 1951 | 5550. | 3.0 | 33.0 | 37 | 34.0 | 5550. | 3.0 | 3170. | 1. | 20. | A | 20. | 1951 |
| 4 | 1952 | 3171. | 36.0 | 14.0 | 38 | 2.0 | 3171. | 36.0 | 3140. | 1. | 20. | B | 21. | 1952 |
| 4 | 1953 | 3790. | 22.0 | 18.0 | 39 | 17.0 | 3790. | 22.0 | 3111. | 1. | 20. | A | 22. | 1953 |
| 4 | 1954 | 3110. | 40.0 | 20.0 | 40 | 0.0 | 3110. | 40.0 | 3110. | 2. | 21. | B | 23. | 1954 |

TESTS FOR INDEPENDENCE, TREND, HOMOGENEITY AND RANDOMNESS

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ANNUAL MAXIMUM DAILY FLOW SERIES 1915 TO 1980 DRAINAGE AREA= 89600.0 SQ.MI.

SUBSAMPLE 2 SAMPLE SIZE = 30

| MONTH | YEAR | DATA | RANK | DI | SEQUENTIAL YEAR NO. | DT | DATA | RANK | ORDERED | FROM SUBSAMPLE | RUNS | ABOVE OR BELOW MEDIAN | RUNAB | YEAR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 1955 | 3821. | 20.0 | 27.0 | 41 | 21.0 | 3821. | 20.0 | 3090. | 1. | 22. | A | 24. | 1955 |
| 6 | 1956 | 2790. | 47.0 | 19.0 | 42 | 5.0 | 2790. | 47.0 | 3031. | 2. | 23. | B | 25. | 1956 |
| 7 | 1957 | 3480. | 28.0 | 34.0 | 43 | 15.0 | 3480. | 28.0 | 3030. | 1. | 24. | A | 26. | 1957 |
| 4 | 1958 | 2170. | 62.0 | 9.0 | 44 | 18.0 | 2170. | 62.0 | 2971. | 1. | 24. | B | 27. | 1958 |
| 5 | 1959 | 2491. | 53.0 | 52.0 | 45 | 8.0 | 2491. | 53.0 | 2970. | 2. | 25. | B | 27. | 1959 |
| 5 | 1960 | 5800. | 1.0 | 64.0 | 46 | 45.0 | 5800. | 1.0 | 2820. | 2. | 25. | A | 28. | 1960 |
| 4 | 1961 | 1890. | 65.0 | 31.0 | 47 | 18.0 | 1890. | 65.0 | 2790. | 2. | 25. | B | 29. | 1961 |
| 5 | 1962 | 3231. | 34.0 | 30.0 | 48 | 14.0 | 3231. | 34.0 | 2770. | 2. | 25. | B | 29. | 1962 |
| 4 | 1963 | 1950. | 64.0 | 3.0 | 49 | 15.0 | 1950. | 64.0 | 2750. | 2. | 25. | B | 29. | 1963 |
| 4 | 1964 | 2210. | 61.0 | 4.0 | 50 | 11.0 | 2210. | 61.0 | 2570. | 1. | 26. | B | 29. | 1964 |
| 5 | 1965 | 2400. | 57.0 | 34.0 | 51 | 6.0 | 2400. | 57.0 | 2560. | 1. | 26. | B | 29. | 1965 |
| 4 | 1966 | 3770. | 23.0 | 6.0 | 52 | 29.0 | 3770. | 23.0 | 2550. | 1. | 26. | A | 30. | 1966 |
| 5 | 1967 | 4110. | 17.0 | 32.0 | 53 | 36.0 | 4110. | 17.0 | 2491. | 2. | 27. | A | 30. | 1967 |
| 4 | 1968 | 2750. | 49.0 | 3.0 | 54 | 5.0 | 2750. | 49.0 | 2490. | 1. | 28. | B | 31. | 1968 |
| 5 | 1969 | 2820. | 46.0 | 2.0 | 55 | 9.0 | 2820. | 46.0 | 2440. | 1. | 28. | B | 31. | 1969 |
| 5 | 1970 | 2770. | 48.0 | 29.0 | 56 | 8.0 | 2770. | 48.0 | 2401. | 1. | 28. | B | 31. | 1970 |
| 5 | 1971 | 3910. | 19.0 | 5.0 | 57 | 38.0 | 3910. | 19.0 | 2400. | 2. | 29. | A | 32. | 1971 |
| 5 | 1972 | 4280. | 14.0 | 12.0 | 58 | 44.0 | 4280. | 14.0 | 2310. | 1. | 30. | A | 32. | 1972 |
| 4 | 1973 | 3540. | 26.0 | 19.0 | 59 | 33.0 | 3540. | 26.0 | 2260. | 1. | 30. | A | 32. | 1973 |
| 5 | 1974 | 4810. | 7.0 | 25.0 | 60 | 53.0 | 4810. | 7.0 | 2250. | 1. | 30. | A | 32. | 1974 |
| 5 | 1975 | 3280. | 32.0 | 11.0 | 61 | 29.0 | 3280. | 32.0 | 2210. | 2. | 31. | A | 32. | 1975 |
| 4 | 1976 | 3820. | 21.0 | 24.0 | 62 | 41.0 | 3820. | 21.0 | 2170. | 2. | 31. | A | 32. | 1976 |
| 4 | 1977 | 2970. | 45.0 | 3.0 | 63 | 18.0 | 2970. | 45.0 | 2010. | 1. | 32. | B | 33. | 1977 |
| 5 | 1978 | 3031. | 42.0 | 38.0 | 64 | 22.0 | 3031. | 42.0 | 1950. | 2. | 33. | B | 33. | 1978 |
| 5 | 1979 | 5410. | 4.0 | 29.0 | 65 | 61.0 | 5410. | 4.0 | 1890. | 2. | 33. | A | 34. | 1979 |
| 4 | 1980 | 3270. | 33.0 | 0.0 | 66 | 33.0 | 3270. | 33.0 | 1630. | 1. | 34. | A | 34. | 1980 |

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                         HISTOGRAM OF THE DATA SAMPLE BY MONTH OF OCCURRENCE

          OTTAWA RIVER AT CHAT FALLS DAM STN. NO. 02KF009                          SAMPLE SIZE N= 66

                         NUMBER OF OCCURRENCES
          0.       10.       20.       30.       40.       50.       60.       70.       80.       90.      100.
     JAN  .         .         .         .         .         .         .         .         .         .         .   0
     FEB  .         .         .         .         .         .         .         .         .         .         .   0
     MAR  .         .         .         .         .         .         .         .         .         .         .   0
     APR  .XXXXXXXXXXXXXXXXXXXXXXXXX              .         .         .         .         .         .         .  25
     MAY  .XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX               .         .         .         .         .         .  34
     JUN  .XXXX     .         .         .         .         .         .         .         .         .         .   4
     JUL  .XX       .         .         .         .         .         .         .         .         .         .   2
     AUG  .         .         .         .         .         .         .         .         .         .         .   0
     SEP  .         .         .         .         .         .         .         .         .         .         .   0
     OCT  .         .         .         .         .         .         .         .         .         .         .   0
     NOV  .X        .         .         .         .         .         .         .         .         .         .   1
     DEC  .         .         .         .         .         .         .         .         .         .         .   0
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## TESTS FOR INDEPENDENCE, TREND, HOMOGENEITY AND RANDOMNESS

OTTAWA RIVER AT CHAT FALLS DAM STN. NO. 02KF009
ANNUAL MAXIMUM DAILY FLOW SERIES 1915 TO 1980   DRAINAGE AREA= 89600.0 SQ.MI.

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SPEARMAN RANK ORDER SERIAL CORRELATION COEFF. FOR INDEPENDENCE=-.1727   D.F.= 63   STUDENTS T=-1.391
                            FROM TABLE CRITICAL T VALUE AT 5 PERCENT SIGNIFICANCE LEVEL TCR.05 = 1.670   NOT SIGNIFICANT
                              -    -     -   -   -   -  1    -        -          -   TCR.01 = 2.388   NOT SIGNIFICANT

SPEARMAN RANK ORDER CORRELATION COEFF. FOR TREND              =-.0250   D.F.= 64   STUDENTS T=-0.200
                            FROM TABLE CRITICAL T VALUE AT 5 PERCENT SIGNIFICANCE LEVEL TCR.05 = 1.999   NOT SIGNIFICANT
                              -    -     -   -   -   -  1    -        -          -   TCR.01 = 2.657   NOT SIGNIFICANT

MANN-WHITNEY SPLIT SAMPLE TEST FOR HOMOGENEITY                       N1= 36   N2= 30   Z=-0.180
                            FROM TABLE CRITICAL Z VALUE AT 5 PERCENT SIGNIFICANCE LEVEL ZCR.05 =-1.645   NOT SIGNIFICANT
                              -    -     -   -   -   -  1    -        -          -   ZCR.01 =-2.326   NOT SIGNIFICANT

WALD-WOLFOWITZ SPLIT SAMPLE TEST FOR HOMOGENEITY                            WALD-WOLFOWITZ R =34.
  N1= 36    N2= 30   FOR THIS SPLIT  Z =-0.057   IS LESS THAN OR EQUAL TO Z AT 5 PERCENT LEVEL = 1.645   NOT SIGNIFICANT

RUNS ABOVE AND BELOW THE MEDIAN TEST FOR GENERAL RANDOMNESS                            RUNAB =34.
  N1= 33    N2= 33    FOR THIS TEST   Z = 0.000   IS LESS THAN OR EQUAL TO Z AT 5 PERCENT LEVEL = 1.960   NOT SIGNIFICANT
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